OPERATOR-SPLIT COMPUTATION OF 3-D SYMMETRIC FLOW

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Abstract

A finite-difference approximation for the 3-D fluid dynamical equations in Cartesian coordinates is obtained by operator-splitting based on the 1-D GRP scheme. To check the accuracy of the 3-D operator-split approximation, two test problems of a symmetrically converging flow are considered, each having an exact solution. In one, the fluid is isentropically and smoothly compressed, whereas in the other the fluid is compressed by a (spherical) stopping shock wave. In the smooth flow test case, the 3-D results agree very well with the exact solution. In the shock compression case the 3-D results are found to agree "in the mean" with the exact solution, although the post-shock flow is somewhat "noisier" than the corresponding 1-D (GRP) solution.

Key Words: operator splitting, symmetric flow, conservation laws scheme

1 INTRODUCTION

The Euler equations that govern the time-dependent flow of an inviscid fluid are routinely solved by conservation laws schemes, such as the GRP scheme. This finite-difference scheme is formulated in one space variable, and has been demonstrated to produce high-resolution solutions to shock wave phenomena (see [1–5]). For the three-dimensional Euler equations in Cartesian coordinates, finite-difference approximations are commonly obtained by applying a one-dimensional scheme in the framework of operator-splitting.

Operator splitting is based on analytic arguments proposed by Strang [6], who showed that judicious sequencing of one-dimensional schemes having a second-order accuracy level, produces a second-order accurate integration of the corresponding multi-dimensional system. However, these arguments are valid only for smooth flows, and not for flows involving discontinuities. To demonstrate the accuracy of 3-D schemes based on operator-split sequencing of the 1-D GRP solver, we consider two test problems of a symmetrically converging flow. In the first the fluid is compressed *smoothly*, while in the second, a uniformly converging fluid is brought to rest by a *shock wave*. Both problems are due to Noh [7], and both have an exact elementary solution. In the second problem, however, Noh's exact solution is restricted to an initially cold gas (zero pressure and finite density). We have extended that problem (and its exact solution) to the case of a finite initial pressure, avoiding the consideration of a non-physical infinitely strong shock.

The primary observation in this study, is that for a smooth flow the 3-D operator-split computations agree very well with the exact solution, whereas for shock compression the results are somewhat noisy, though "in the mean" they agree well with the exact solution.

We start with a description of the operator splitting scheme, and an outline of the symmetric test problems. This is followed by a presentation and discussion of the computation results, ending with some concluding remarks.

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